Work, Energy and Power Cheat Sheet

In this chapter, we will learn to solve problems regarding the motion of a particle by considering its energy. This chapter is split up into three parts: In the first, we will discuss kinetic and gravitational potential energies and learn about the idea of "work done". We will then look at how we can use the conservation of energy and work-energy principle to solve more complicated problems. Finally, we will learn to calculate the power produced by an engine and use this to solve problems regarding the motion of a moving vehicle.

Kinetic and potential energy

At any given time, a particle possesses only two types of energy: kinetic and potential (also called gravitational potential energy, or G.P.E for short)

- The kinetic energy ($K.E.$) of a particle is given by $K.E = \frac{1}{2}mv^2$, where m is its mass and v its speed.
- The potential energy ($P.E.$) of a particle is given by $P.E. = mgh$, where h is the height of the particle above a fixed zero level.

Since both of the above quantities represent energy, they are both measured in Joules (J).

Remember that when calculating the potential energy of a particle, you must choose a zero level (i.e. a fixed level where $P.E = 0$). This can be anywhere, but it is conventional to choose the zero level to be the lowest point in the particle's motion. Be aware that potential energy can be negative, if the particle falls below the zero level.

Work done

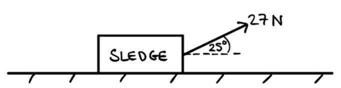
Work done is simply just the transfer of energy from one object to another. We say that work is done on an object when a force causes it to move a particular distance. To calculate the work done by a force F on an object:

- $Work\ Done = \begin{bmatrix} component\ of\ force\ in \\ direction\ of\ motion \end{bmatrix} \times \begin{bmatrix} distance\ moved\ in \\ direction\ of\ motion \end{bmatrix}$

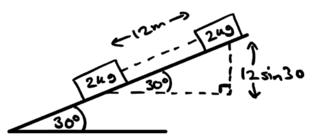
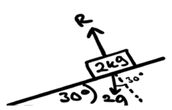
You may sometimes be asked to calculate the "work done against gravity", when a particle is raised vertically.

- The work done against gravity when a particle is raised vertically is equal to its change in potential energy. In other words, **work done against gravity** $= mgh$, where h is the height raised.

Example 1: A sledge is pulled 15 m across a smooth sheet of ice by a force of magnitude 27 N. The force is inclined at 25° to the horizontal. By modelling the sledge as a particle, calculate the work done by the force.

We start with a detailed diagram.	
The box moves across the ice, so the work done is going to be equal to **the horizontal component** of the force multiplied by the distance travelled.	$Work\ done = (27\cos 25) \times 12 = 293.6\ J$

Example 2: A package of mass $2kg$ is pulled at a constant speed up a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between the package and the surface is 0.35. The package is pulled $12m$ up a line of greatest slope of the plane. Calculate: a) the work done against gravity. b) the work done against friction.

a) We start with a detailed diagram.	
Work done against gravity $= mgh$	$Work\ done\ against\ gravity = (2)(g)(12\sin 30)$ $= 12g\ N$
b) Use work done = force × distance	$Work\ done\ by\ friction = F \times d$ $= (0.35)R(12)$
We resolve perpendicular to the plane to find R.	$R - 2g\cos 30 = 0$ $\therefore R = 2g\cos 30$ 
Calculate work done.	$Work\ done = (0.35)(2g\cos 30)(12) = 71.3\ J$

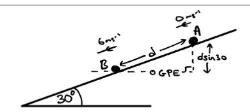
Conservation of energy

The principle of conservation of energy states that:

- When no external forces (besides gravity) do work on a particle during its motion, the total energy possessed by the particle remains constant.

This idea is useful when tackling problems where **no external forces act on a particle**.

Example 3: A smooth plane is inclined at 30° to the horizontal. A particle of mass 0.5 kg slides down a line of greatest slope of the plane. The particle starts from rest at point A and passes point B with a speed 6 ms^{-1}. Find the distance AB.

We start again with a detailed diagram. Taking B to be the zero P.E level.	
We find the kinetic and potential energy of the particle at A, then at B.	At A: $K.E = \frac{1}{2}(0.5)(0)^2 = 0\ J$ $P.E = mgh = 0.5g(d\sin 30)$ At B: $K.E = \frac{1}{2}mv^2 = \frac{1}{2}(0.5)(6)^2 = 9\ J$ $P.E = mgh = (0.5)(g)(0) = 0$
Using the conservation of energy, we can use the fact that the total energy at A = total energy at B, since no external forces act on the particle (the plane is smooth).	Total energy at $A = 0.5gd\sin 30 = 0.25gd$ Total energy at $B = 9$ $\therefore 0.25gd = 9$
Rearrange for d.	$d = \frac{9}{0.25g} = 3.67\ m$

Work-energy principle

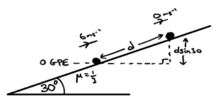
It is of course more realistic to expect a scenario where a particle is subject to an external force, whether that be a resistive force such as friction or perhaps an applied force acting upon the particle. For such problems, the work-energy principle is very useful:

- The change in the total energy of the particle is equal to the work done on the particle.

When approaching questions where an external force acts on a particle, the general procedure is to:

1. Find the total energy of the particle at the beginning and end of the described motion.
2. Find the change in energy, by subtracting the final energy from the initial energy.
3. Find the work done by the external force on the particle.
4. Equate this to the change in energy and solve for any unknown constants.

Example 4: A box of mass 2 kg is projected with speed 6 ms^{-1} up a line of greatest slope of a rough plane inclined at 30° to the horizontal. The coefficient of friction between the box and the plane is $\frac{1}{3}$.
a) Use the work-energy principle to calculate the distance the box travels up the plane before coming to rest.
b) Suggest why in practice the box may not travel as far as the distance you calculated.

a) We start again with a detailed diagram. Take the starting point to be the zero P.E level. Let d represent the distance travelled up the plane till the point of instantaneous rest.	
We find the kinetic and potential energy of the particle at A, then at B.	At start: $K.E = \frac{1}{2}(2)(6)^2 = 36\ J$ $P.E = mgh = 2g(0) = 0$ At end: $K.E = \frac{1}{2}mv^2 = \frac{1}{2}(2)(0)^2 = 0\ J$ $P.E = mgh = (2)(g)(d\sin 30) = gd$
Next, find the change in energy and the work done by the frictional force. Remember that change in energy = energy before – energy after. We also use $F_{max} = \mu R$ since friction is limiting.	$\Delta Energy = 36 - gd$ Work done by resistive force $= Force \times distance$ $= F \times d$ $= \mu R \times d = \frac{1}{3}Rd$
Find R by resolving perpendicular to the plane:	$R - 2g\sin 30 = 0$ $\therefore R = 2g\sin 30 = g$
So, the work done by the frictional force becomes:	$\therefore$ work done $= \frac{1}{3}gd$
Use the work-energy principle.	$\Delta Energy = Work\ Done\ by\ friction$ $\Rightarrow 36 - gd = \frac{1}{3}gd$
Solve for d.	$d\left(g + \frac{1}{3}\right) = 36 \Rightarrow d = \frac{36}{g + \frac{1}{3}} = 3.55m$
b) This is a common answer to such questions.	In reality air resistance is likely to oppose the motion of the particle causing it to travel a shorter distance than the one we found.

Power

Power is defined as the rate of doing work. When considering problems involving the motion of a moving vehicle:

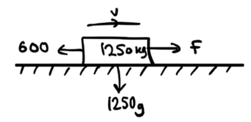
- You need to model the driving force of the vehicle, when drawing force diagrams.

You will also need to make use of the following relationship between the power produced by the engine and the driving force:

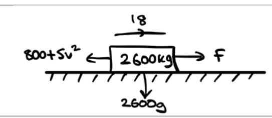
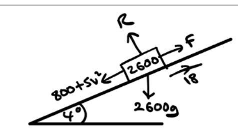
- $P = Fv$, where P is the power of the engine, F is the driving force and v is the speed of the vehicle.

Power is measured in watts (W). Questions will often state the power of a vehicle in kilowatts (kW). In such cases you need to make sure you use watts in your working. Remember that 1 kW = 1000 W

Example 5: A van of mass 1250 kg is travelling along a horizontal road. The van's engine is working at 24 kW. The constant resistance to motion has magnitude 600 N. Calculate:
a) the acceleration of the van when it is travelling at 6 ms^{-1}.
b) the maximum speed of the van.

a) We start with a detailed diagram.	
Use $P = Fv$.	$24000 = Fv$ At $v = 6$, $24000 = F(6)$ $\therefore F = \frac{24000}{6} = 4000$ N
Use $F = ma$.	$F - 600 = 1250(a)$ $4000 - 600 = 1250(a)$
Solve for a.	$\Rightarrow a = \frac{3400}{1250} = 2.72\ ms^{-2}$
b) We again use $P = Fv$, but we make F the subject.	$24000 = Fv$ $\therefore F = \frac{24000}{v}$
Use $F = ma$.	$F - 600 = 1250(a)$ $\frac{24000}{v} - 600 = 1250(a)$
But note that at the maximum speed, $a = 0$.	$\frac{24000}{v} - 600 = 0$
Rearrange for v.	$24000 = 600v$ $\therefore v = \frac{24000}{600} = 40\ ms^{-1}$

Example 6: A car of mass 2600 kg is travelling in a straight line. At the instant when the speed of the car is v ms^{-1}, the total resistances to motion are modelled as a variable force of magnitude $(800 + 5v^2)$ N. The car has a cruise control feature which adjusts the power generated by the engine to maintain a constant speed of 18 ms^{-1}. Find the power generated by the engine when:
a) the car is travelling on a horizontal road.
b) the car is travelling up a road that is inclined at an angle 4° to the horizontal.

a) We start with a detailed diagram.	
Use $F = ma$	$F - (800 + 5v^2) = 2600(a)$
We are told $v = 18\ (constant)$, and thus $a = 0$. Substitute in these values.	$F - (2420) = 2600(0)$ $F = 2420$
Use $P = Fv$.	$P = Fv$ $P = (2420)(18) = 43\,560\ W = 43.6\ kW\ (3\ s.f.)$
b) We draw another detailed diagram.	
Use $F = ma$ up the slope.	$F - 2600g\sin 4 - (800 + 5v^2) = 2600a$
We are told $v = 18\ (constant)$, and thus $a = 0$. Substitute in these values.	$F - 10400g\sin 4 - 2420 = 0$ $\therefore F = 9530$ N
Use $P = Fv$.	$P = Fv$ $P = (9530)(18) = 171\,532.. = 170\ kW\ (3\ s.f.)$